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Dynamic calibration of inclined and crossed hot wires

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Single inclined hot wires and X-wires are calibrated directly for absolute turbulence intensity measurements by oscillating the wire probe sinusoidally with accurately known motion in a steady stream. The usual static method of calibration shows serious discrepancies and uncertainties when compared with this new procedure. The new method also provides a check on the small perturbation linearity of inclined wires.

1. Introduction

The difficulties associated with the static calibration of hot wires are well known. Static calibrations involve differentiation of discrete and usually scattered data in order to obtain the system sensitivities. The differentiation is usually done by estimating the local derivative of a curve of best fit to the data, and considerable uncertainties are introduced. There are a variety of empirical laws available for fitting to the calibration data but they all give different answers for the hot-wire sensitivity. When inclined hot wires are calibrated statically the wire angle must also be known accurately. This is difficult since the wire is invariably bowed owing to thermal expansion. The most commonly used calibration method is due to Hinze (1959) and this was later developed by Champagne, Sleicher & Wehrmann (1967). This method introduces further uncertainties owing to the difficulty in obtaining a reliable value for the additional quantity k which is introduced to include the effect of longitudinal cooling. Comparisons of published measurements of k show substantial scatter; see, for example, Webster (1962), Champagne *et al.* (1967) and Guitton & Patel (1969).

Perry & Morrison (1971) found substantial errors in the conventional static calibration procedure for constant-temperature hot wires held normal to the bias velocity. They developed an alternative procedure which avoided the need to specify the functional form of the heat-transfer law and the need to differentiate the calibration curve graphically or numerically. This alternative procedure involves shaking the hot wire with accurately known sinusoidal velocity perturbations at low frequency. The superiority of this 'dynamic' method over the conventional static technique was demonstrated by a series of direct tests. In this paper the dynamic method is extended to cover calibration of inclined hot wires and X-wires.

2. Dynamic small-perturbation calibration of single inclined hot wires

The details of the dynamic-calibration apparatus and the results of the calibration procedure for normal hot wires are described by Perry & Morrison (1971).

If an inclined hot wire is oscillated sinusoidally in a steady flow along the flow direction, the longitudinal sensitivity $\partial E_0/\partial U$ may be determined (E_0 being the hot-wire output voltage). If it is oscillated across the mean flow, the transverse sensitivity $\partial E_0/\partial V$ may be evaluated. The wire must, of course, have identical inclinations to the mean flow direction in each test. The Reynolds-stress sensitivity of an inclined wire may be determined directly by oscillating the hot wire along a path inclined to the mean flow direction and in the plane parallel to the mean flow and parallel to the hot-wire axis. Thus the wire experiences simultaneous longitudinal (u') and transverse (v') velocity perturbations. The small-perturbation output voltage of a wire inclined at an angle α to the mean flow is given by

$$e_{0_1} = K_1 u' + K_2 v', \tag{1}$$

where $$K_1 = \partial E_0/\partial U \quad \text{and} \quad K_2 = \partial E_0/\partial V.$$

If the wire angle is changed to $-\alpha$ by rotating the wire about an axis parallel to the mean flow direction the output voltage perturbation is

$$e_{0_2} = K_1 u' - K_2 v'. \tag{2}$$

A combination of the mean-square values of equations (1) and (2) gives

$$\overline{e_{0_1}^2} - \overline{e_{0_2}^2} = 4K_1K_2\overline{u'v'}. \tag{3}$$

Thus if the two output voltages are measured for an accurately known velocity field $\overline{u'v'}$ the Reynolds-stress sensitivity is

$$K_3 = [\overline{e_{0_1}^2} - \overline{e_{0_2}^2}]/\overline{u'v'}. \tag{4}$$

Equation (3) also indicates that the Reynolds-stress sensitivity is related to the longitudinal and transverse sensitivities as follows:

$$K_3 = 4K_1K_2. \tag{5}$$

This assumes the linearity given by (1) and (2). To the authors' knowledge this assumption has not been conclusively verified experimentally elsewhere. If the above measurements are repeated for a range of bias velocities, the system sensitivity may be determined as a function of the bias velocity. Some typical dynamic calibration results are shown in figures 1 and 2. The probe used was a Disa type 55A29 (inclined) with the tungsten filament replaced by a 4 μm diameter Wollaston-type platinum wire.† The wire angle was 45° and its sensing length was 1·2 mm, obtained by an etching process. Gilmore (1967) and Dahm & Rasmussen (1969) showed that probes similar to this have negligible prong interference when

† Early unpublished work of Perry & Morrison showed that calibrations with tungsten drifted seriously with exposure time. Annealed platinum was consistent to 1% after several hours of use.

the probe stem is approximately aligned with the mean flow, as was the case during all tests reported here. Figure 2 also compares the Reynolds-stress sensitivity obtained by direct measurement of a known 'Reynolds-stress field' relative to the wire generated by shaking the wire along an inclined path at a known amplitude and frequency in a uniform steady stream. The sensitivity was also calculated from the separate transverse and longitudinal sensitivities. Agreement to within 1 % was obtained, thus verifying the assumption of linearity given in (1) and (2).

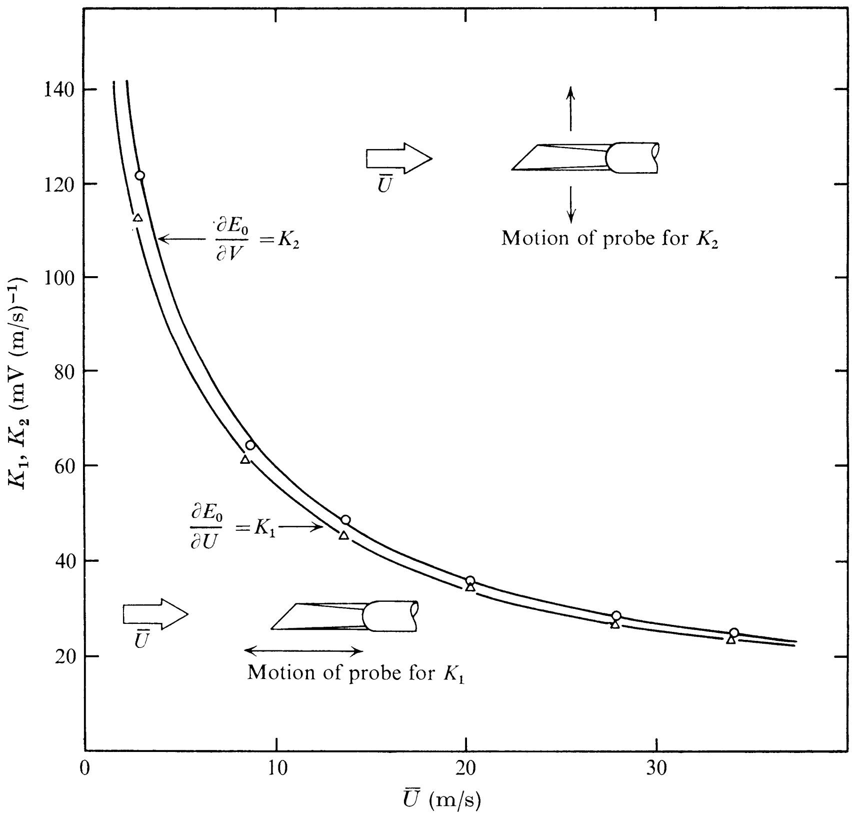

FIGURE 1. Dynamic calibration of hot wire.

The advantage of this technique is that knowledge of the form of the heat-transfer law for inclined or normal hot wires, and the true wire angle are not required. If the static calibration procedure is used, the wire angle must be known with great accuracy. For wire angles near 45° a one degree error in α introduces a 4 % error in the transverse and Reynolds-stress sensitivity. As the response of inclined hot wires is sensitive to changes of wire angle and since most hot wires have a slight bow when heated, a source of error could be introduced in the dynamic calibration if the wire were deflecting and thus changing its inclination to the mean flow. This possible error was investigated by calibrating a hot wire for a range of calibrator frequencies, ω, for a fixed bias velocity. As the calibrator frequency ω changes, the dynamic loading on the wire changes like ω^2. In such an experiment ω was varied from 3·5 to 9 Hz, changing the dynamic loads by a factor of 6·5. No change was observed in the system of sensitivity. For typical bias conditions the inertia loads generated by the calibrator are one or more orders of magnitude less than the static load due to the mean air speed.

The flow in the wind tunnel working section was not disturbed by the oscillation of the probe assembly. This was checked by placing a stationary hot wire in the working section upstream of and close to the oscillating probe. No perturbations in velocity could be detected since they were less than the free-stream turbulence level of 0·2 %, while typical signal levels from the oscillating probe were 10 % of the mean velocity (zero to peak).

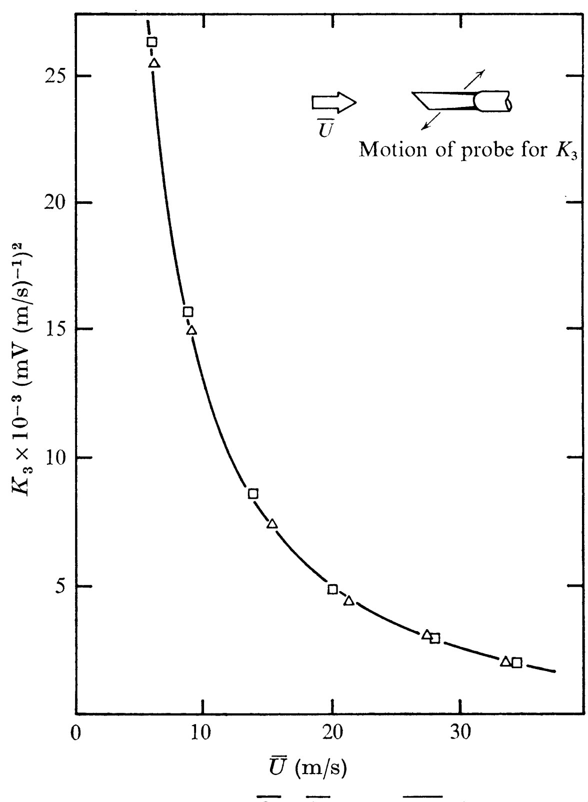

FIGURE 2. Reynolds-stress sensitivity. $\overline{e^2_{0_1}} - \overline{e^2_{0_2}} = K_3\overline{u'v'}$; △, direct calibration in known Reynolds-stress field; □, calculation of K_3 from independent u' and v' calibrations.

3. Comparison of conventional and dynamic calibration of inclined hot wires

The conventional static methods are typified by the procedure adopted by Champagne *et al.* (1967) and this will now be compared with the dynamic technique. There are many variations of the static method, e.g. Friehe & Schwarz (1968), Davis & Bruun (1968) and Bruun (1971), but these were not tested.

Champagne's method for evaluating the system sensitivity is the same for both inclined and normal wires since the same heat-transfer law is used. That is, for a wire held normal to the flow he takes

$$E_0^2 = A + BU^n, \tag{6}$$

where A and B are constant for a given fluid, wire, electronic circuit and wire temperature. The index n lies between 0·4 and 0·5, depending on the form of the law favoured by the operator. The small-perturbation longitudinal sensitivity for a normal hot wire may be obtained from (6) as follows:

$$\partial E_0/\partial U = nB/2\bar{E}_0\bar{U}^{1-n}, \tag{7}$$

where the overbars denote temporal means. Thus to evaluate $\partial E_0/\partial U$ the gradient B of the calibration data plotted on an E_0^2 *vs.* U^n set of axes must be determined. This is usually achieved by estimating the straight line of best fit on an E_0^2 *vs.* U^n plot. If the functional form of the static response of an inclined hot wire to normal and axial velocity components is known the transverse sensitivity $\partial E_0/\partial V$ may be calculated from a modified form of (6). The scheme proposed by Hinze (1959) and Champagne *et al.* (1967) makes use of an effective cooling velocity U_e in place of the velocity U in (6). This is given by

$$U_e^2 = U_N^2 + k^2U_L^2, \tag{8}$$

where k is the longitudinal cooling factor and U_N and U_L are the velocity components normal and parallel to the inclined hot wire. If there are longitudinal (U) and transverse (V) components of mean velocity, (8) yields

$$U_e^2 = (U\cos\alpha + V\sin\alpha)^2 + k^2(U\sin\alpha - V\cos\alpha)^2. \tag{9}$$

If U_e is substituted into (6) the transverse sensitivity may be obtained:

$$\left.\frac{\partial E_0}{\partial V}\right|_{\bar{V}=0} = \frac{nB'}{2\bar{E}_0\bar{U}^{1-n}}\,\frac{\tan\alpha\,(1-k^2)}{1+k^2\tan^2\alpha} = K_2. \tag{10}$$

Also
$$\left.\frac{\partial E_0}{\partial U}\right|_{\bar{V}=0} = \frac{nB'}{2\bar{E}_0\bar{U}^{1-n}} = K_1, \tag{11}$$

where
$$B' = B\cos^n\alpha[1+k^2\tan^2\alpha]^{\frac{1}{2}} \tag{12}$$

and B' is obtained directly from a plot of E_0^2 *vs.* U^n for the wire held in the inclined position. The Reynolds-stress sensitivity may then be determined from (5), thus

$$K_3 = 4\left[\frac{nB'}{2\bar{E}_0\bar{U}^{1-n}}\right]^2\frac{\tan\alpha(1-k^2)}{1+k^2\tan^2\alpha}. \tag{13}$$

Figures 3, 4 and 5 compare the various sensitivities as determined from the static and dynamic calibration methods. For the static calibration, Champagne's values of n and k were used ($n = 0{\cdot}5, k = 0{\cdot}2$) and from an enlarged shadow of the wire the angle was estimated to be 45°. The results demonstrated that substantial errors exist in the static calibration procedure for inclined hot wires. The errors in the evaluation of the longitudinal sensitivity of inclined wires are due to the same factors as those outlined by Perry & Morrison (1971) for the calibration of normal wires. The major error is due to the judgement the operator must use when fitting a straight line to the calibration data, which invariably has slight curvature and experimental scatter. There may be additional errors in the

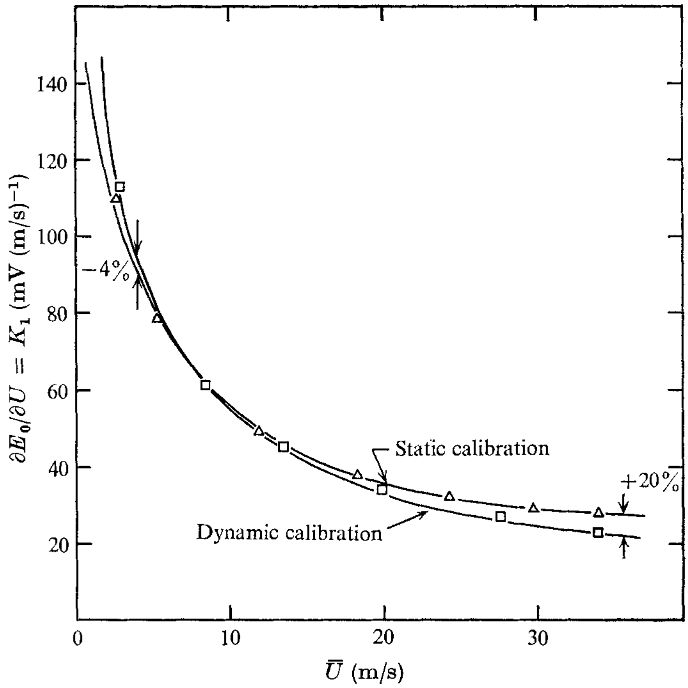

FIGURE 3. Comparisons of inclined wire calibrations for longitudinal sensitivity.

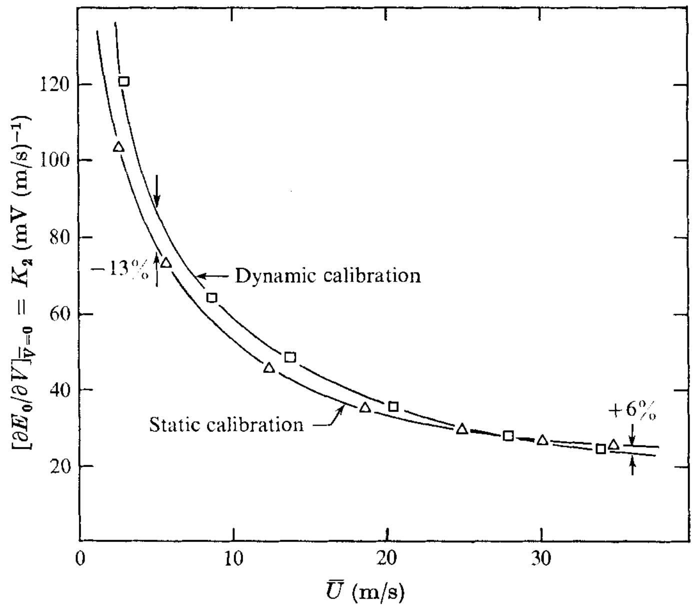

FIGURE 4. Comparison of inclined wire calibrations for transverse sensitivity.

evaluation of the transverse sensitivity owing to errors in the measurement of the wire angle. Also the functional form of the heat-transfer law is not described in sufficient detail by the normal heat-transfer law (6), see Perry & Morrison (1971), let alone a modified version.

To check that the static results used in this report were close to those obtained by Webster (1962) and Champagne *et al.* (1967) a method similar to theirs was used to evaluate the longitudinal cooling factor k. The King's law form of the static calibrations of a hot wire at angles of $\alpha = 0$ and $45°$ is shown in figure 6. From the gradients of these two calibration lines and (6) and (8), k was found to

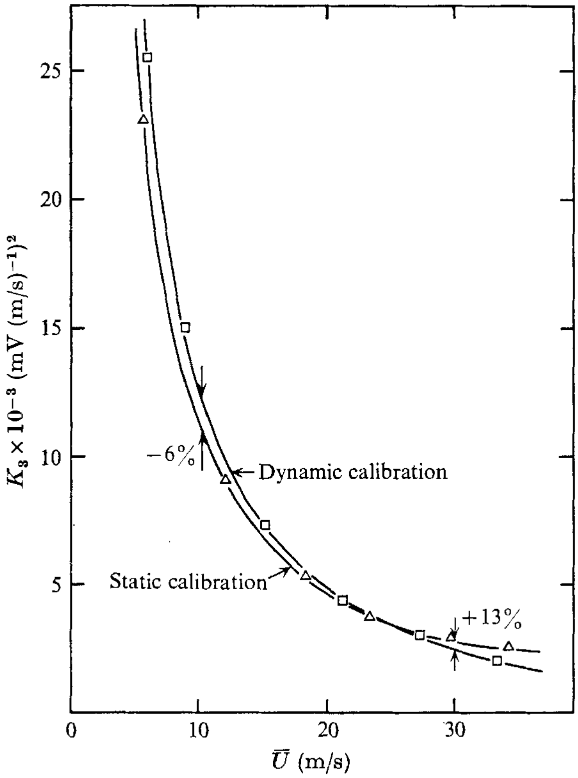

FIGURE 5. Comparison of inclined wire calibrations for Reynolds-stress sensitivity.

be 0·26, which is of the same order as the value given by the above workers. Thus the authors' static results coincide with previous published results when used for predicting k. However, the more accurate dynamic evaluations of the small perturbation sensitivities differ from those determined by the static calibration procedure.

The static calibration for Reynolds stress is fairly insensitive to the value of k chosen (e.g. Reynolds stress varies by 13 % for k varying from 0 to 0·2). If n is taken as 0·4 instead of 0·5, k becomes 0·36 and the Reynolds stress predicted by static calibrations increases by 10 %. This serves to illustrate the uncertainties in the static method.

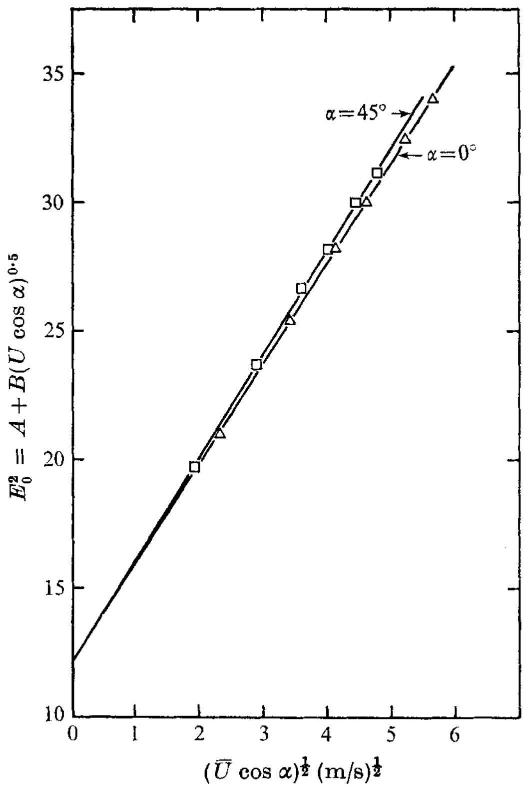

FIGURE 6. Static calibration for determining the factor k.

4. X-wire calibration

It can be shown that a single inclined hot wire which can be rotated about the probe axis is suitable for measuring Reynolds stress only. To measure u' and v' as well as $\overline{u'v'}$ it is necessary to use X-wires. Basically, the principles involved in the calibration of X-wires are the same as for single inclined wires. However, a comparison between a dynamically calibrated single inclined hot wire and an X-wire shows the latter to be less accurate for Reynolds-stress measurements because of the larger number of sensitivity coefficients which must be known. This difficulty is also encountered in static calibrations since here, also, X-wires require more constants to be known than in the single inclined wire case.

To investigate the performance of X-wires the authors carried out the following tests. A Disa X-wire probe (type 55A38 miniature) was modified by increasing the prong separation to reduce interference (Guitton 1968) and the tungsten wire filaments were replaced with the same platinum wire size as mentioned earlier. The wire angles were $\pm 45°$. The two wires of the X-wire array were calibrated dynamically for both transverse and longitudinal sensitivities by oscillating the probe parallel and then transverse to the free stream, thus yielding the sensitivity coefficients K_{11}, K_{12}, K_{21} and K_{22}. The output voltages from the two wires are thus given by

$$e_1 = K_{11}u' + K_{12}v', \tag{14}$$

$$e_2 = K_{21}u' - K_{22}v'. \tag{15}$$

A known Reynolds-stress field was then set up by using the dynamic calibrator to oscillate the probe along a path inclined to the free stream. The two voltage signals were simultaneously fed to an analog computer, which with appropriate scaling yielded

$$\overline{u'v'} = \frac{\overline{e_1^2 K_{21} K_{22} + e_1 e_2 (K_{12} K_{21} - K_{11} K_{22}) + e_2^2 K_{11} K_{12}}}{(K_{12} K_{21} + K_{11} K_{22})^2}. \tag{16}$$

By comparing this equation with (4) and (5) it can be seen that more inaccuracies are introduced in the X-wire method than in the single inclined wire method because extra coefficients must be evaluated and used to scale the computer. However, measured values of $\overline{u'v'}$ using the X-wire array agreed with the known values of Reynolds stress to within $\pm 5\%$ over a mean velocity range of 4–30 m/s. Also, a dynamically calibrated single inclined wire and an X-wire array were used for measuring the Reynolds stress in a turbulent boundary layer. These measurements also agreed to within $\pm 5\%$. For detailed surveys of Reynolds-stress fields in turbulent boundary layers the more accurate dynamically calibrated single inclined wire should be used. A third comparison was made between the measurement of u' in a turbulent boundary layer using a dynamically calibrated normal wire and an X-wire array. The measurements agreed to within $\pm 2\%$.

5. Hot-wire nonlinearity

To avoid nonlinearity errors when using the dynamic calibration technique, it is essential that the calibration signal is only a small perturbation when compared with the free-stream velocity. The effect of large calibration signals on the dynamic calibration was checked by using the calibrator to generate sinusoidal velocity perturbations with r.m.s. values in the range 0–$0{\cdot}4\overline{U}$, while the free-stream velocity $\overline{U}$ was held fixed.

For unidirectional calibration of normal and inclined wires, less than 1 % error occurred in $\partial E_0/\partial U$ or $\partial E_0/\partial V$ provided $u'/\overline{U}$ or $v'/\overline{U}$ had r.m.s. values less than 0·2. It must be noted that during tests where there is a longitudinal or transverse velocity perturbation occurring separately, there are no errors due to velocity component interaction. For Reynolds-stress calibrations simultaneous u' and v' perturbations are used and so both heat-transfer nonlinearities and velocity component interaction are present. Less than 1 % error was observed in $\overline{u'v'}$ provided $u'/\overline{U}$ and $v'/\overline{U}$ both had r.m.s. values less than 0·14. To ensure that nonlinearity errors did not affect the dynamic calibrations the peak calibrator speed was always adjusted to be less than $0{\cdot}1\overline{U}$.

6. Conclusions

The static calibration of inclined hot wires and X-wire arrays has errors similar to those for the calibration of normal hot wires. The conclusions of Perry & Morrison (1971) concerning the calibration of normal hot wires may also be applied to the calibration of inclined hot wires and X-wires. These errors may be overcome by using a small-perturbation calibration technique similar to the one proposed in this paper. The consistency of the dynamic calibration technique was demonstrated by comparing the Reynolds-stress sensitivity as determined from a known Reynolds stress field with the same sensitivity as calculated from the independently determined transverse and longitudinal sensitivities. This test also demonstrates that the longitudinal and transverse responses of an inclined hot wire are linearly independent.

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